

**M U N I**  
**F I**

# **Mornfall's Divine tool and me**

Abstract representation in interval domain

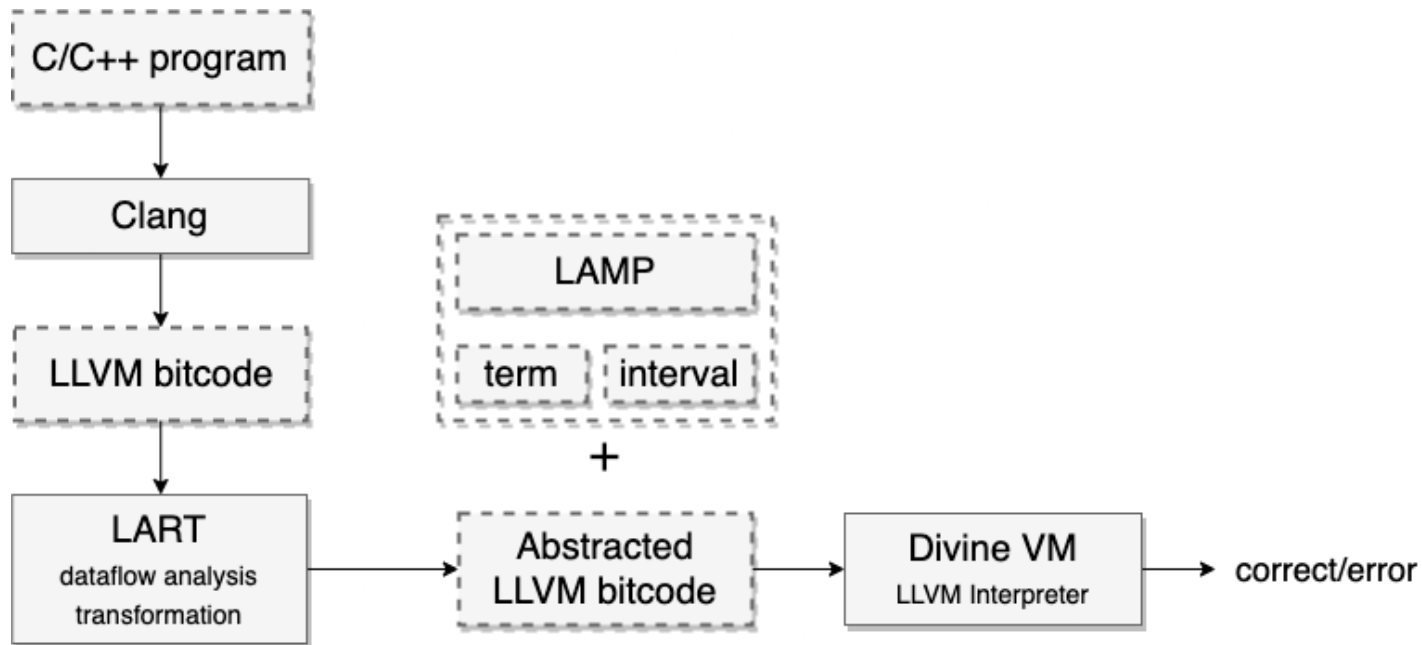
Pavol Mišenko

# Quick recap

How to deal with nondeterminism?

- Consider all possible options
- Symbolic representation
- Abstract domain representation
  - Unit domain
  - Zero domain
  - Sign domain
  - Interval domain

# Analysis workflow



# Problems to solve with interval domain

## 1. Domain representation

- Value representation  $[3, 5] \sim \{3,4,5\}$
- Operations  $[3, 5] + [1, 2] = [4, 7] \sim \{4, 5, 6, 7\}$

## 2. Nondeterministic control flow

## 3. Branch constraint propagation

```
i = [ 4, 8 ]  
if i < 6:  
    ...    i = [ 4, 5 ]  
else  
    ...    i = [ 6, 8 ]
```

# Domain representation

- Interval domain lattice

- Meet -  $\cap$
- Join -  $\cup$

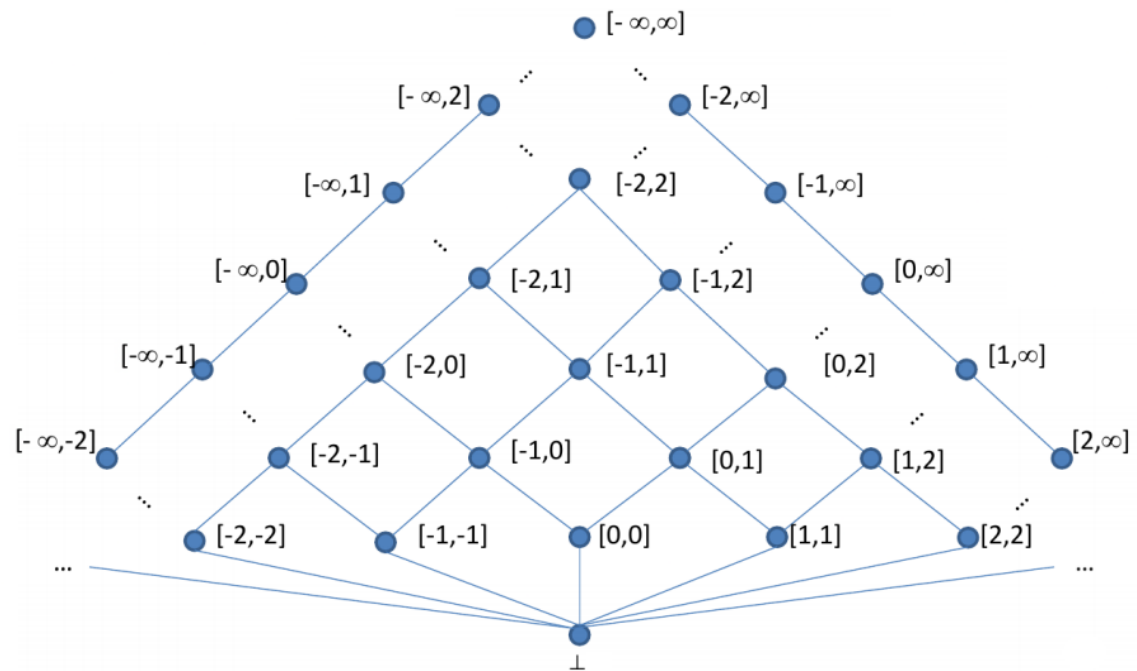
- Nondeterminism

```
int x = input()
```

↓  
 $x = [-\infty, \infty]$

```
int y = 5
```

↓  
 $y = [5, 5]$



# Relational operations - LT

A = [1, 4]

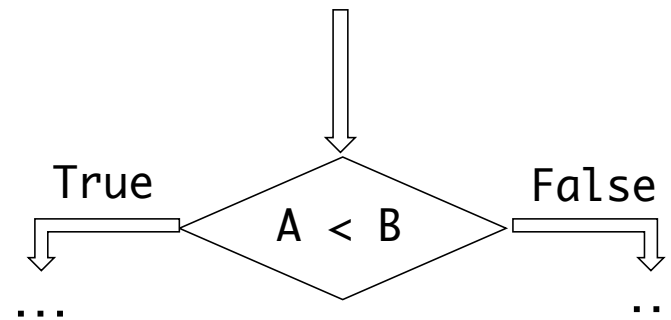
B = [2, 7]

C = [5, 7]

A < C -> True

C < A -> False

A < B -> Maybe



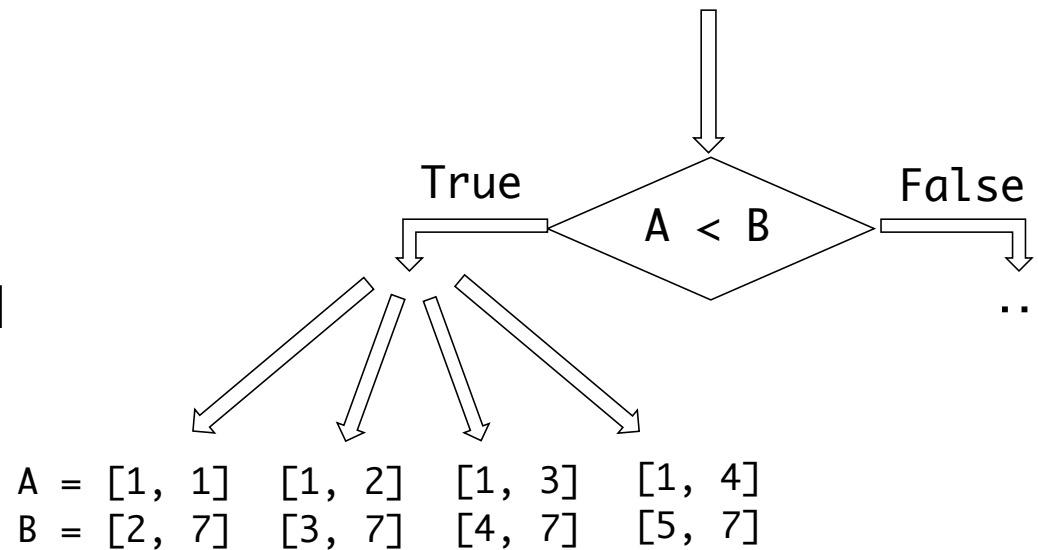
# Relational operations – BOP\_LT

A = [1, 4]

B = [2, 7]

[1, 2, 3, 4]

[2, 3, 4, 5, 6, 7]



# Arithmetic operations – BOP\_ADD

$$A = [2, 5] \quad B = [3, 5]$$

$$R = A + B = [5, 10]$$

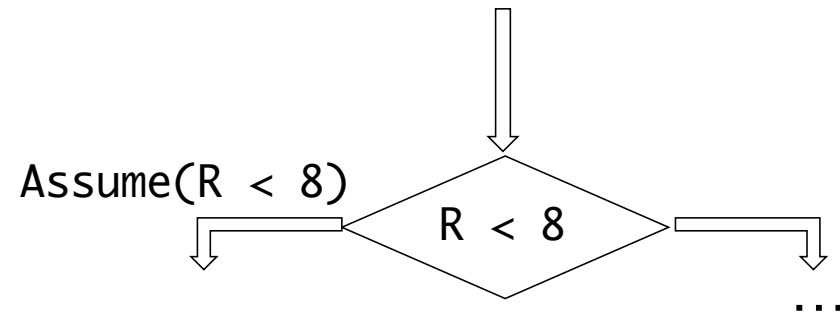
$$R' = [5, 7]$$

Trivial approach:

$$A_T = A \cap (R' - B) = [2, 5] \cap [0, 4] = [2, 4]$$

$$B_T = B \cap (R' - A) = [3, 5] \cap [0, 5] = [3, 5]$$

$$A_T + B_T = R_T = [5, 9]$$





# Arithmetic operations – BOP\_ADD

$$A = [2, 5]$$

$$B = [3, 5]$$

$$A_T = [2, 4]$$

$$B_T = [3, 5]$$

$$R' = [5, 7]$$

$$R_T = [5, 9]$$

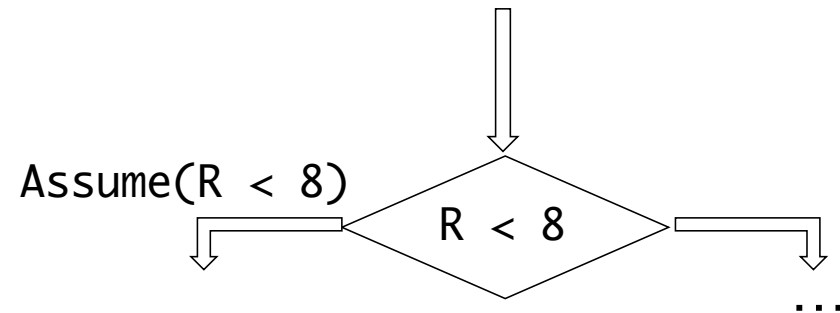
'Choose' approach:

$$\Delta_H = RH_T - RH' = 9 - 7 = 2 \quad \Delta_L = RL' - RL_T = 5 - 5 = 0$$

$$i = \text{choose}(\Delta_H + 1)$$

$$A = [AL_T, AH_T - i]$$

$$B = [BL_T, BH_T - \Delta_H + i]$$



# Arithmetic operations – BOP\_ADD

$$A_T = [2, 4]$$

$$B_T = [3, 5]$$

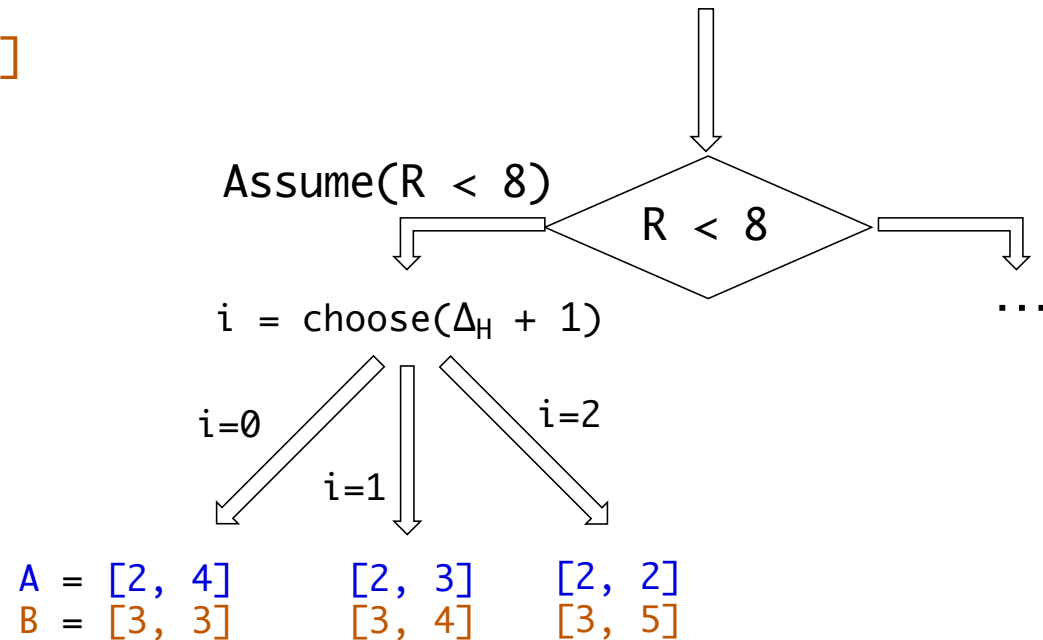
$$\Delta_H = 2$$

$$\Delta_L = 0$$

$$R' = [5, 7]$$

$$A = [AL_T, AH_T - i]$$

$$B = [BL_T, BH_T - \Delta_H + i]$$

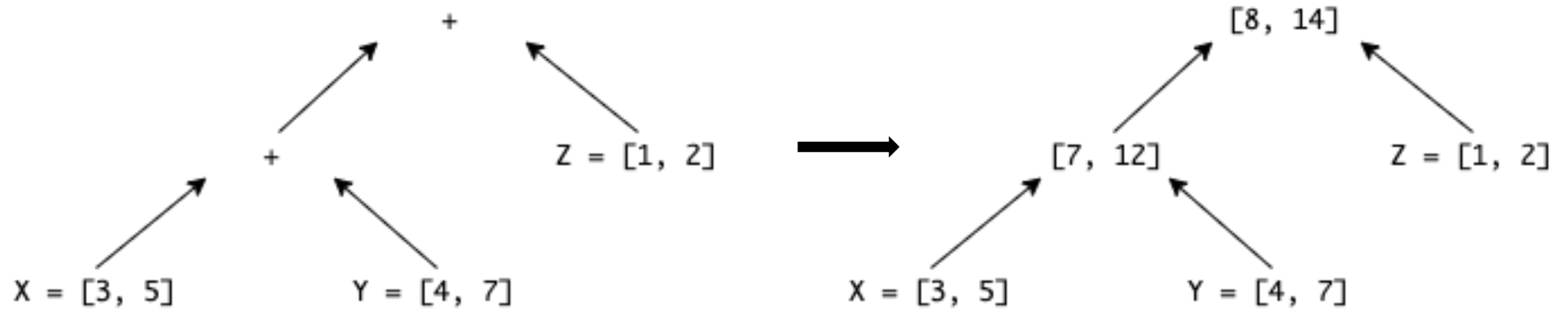


# Arithmetic operations – BOP\_ADD

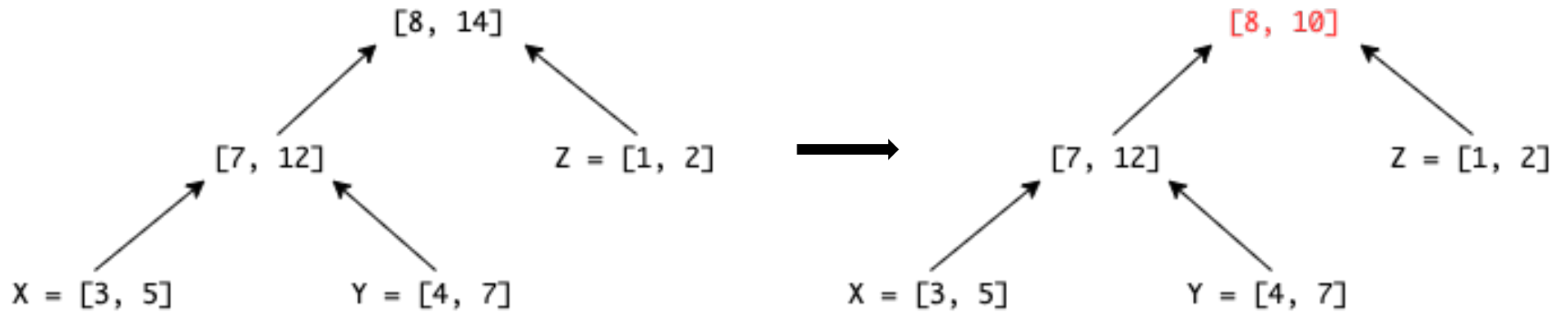
## Invariants:

- Sum of intervals A and B after choose is equivalent to expected R'
- Unity of all intervals A across branches after choose is equivalent to  $A_T$  before choose.  
(Same for B)

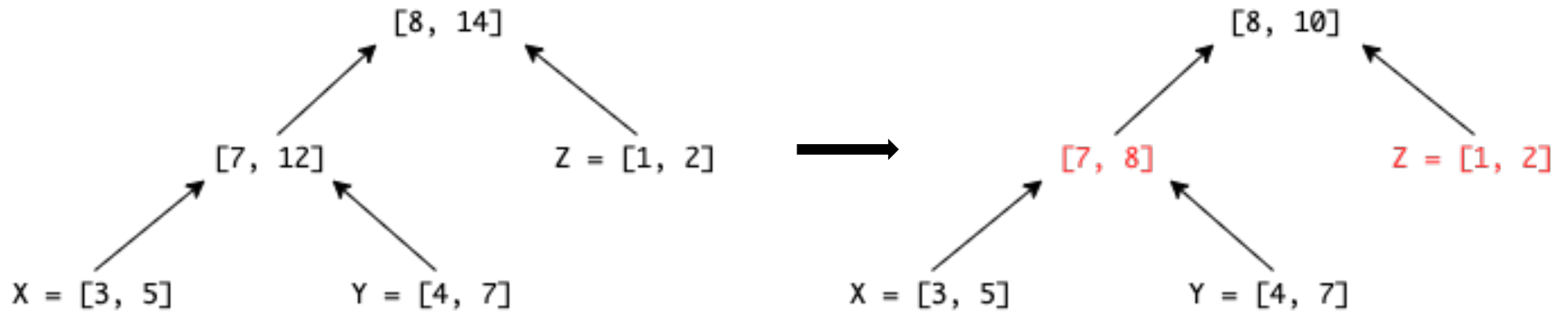
# Multiple level propagation



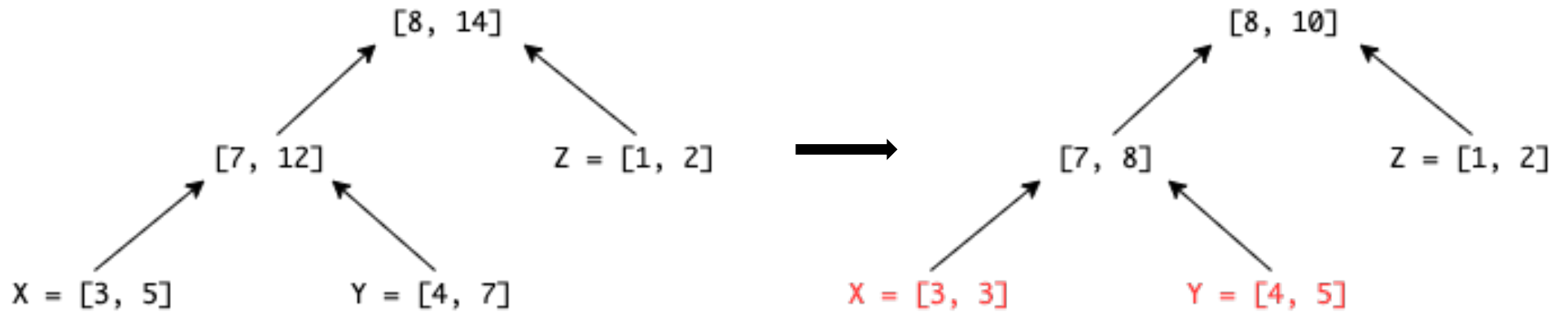
# Multiple level propagation



# Multiple level propagation



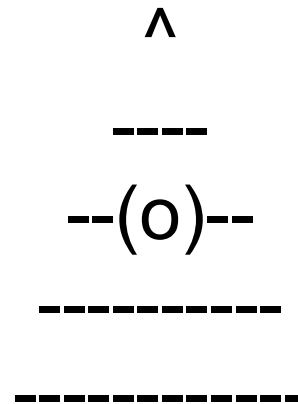
# Multiple level propagation



# What's Next

- Bitwise operation refinement
- Domain refinement
  - BOP splitting
- BOP propagation constraints





**Thank you for attention**